Towards a Local Kalman Filter for Visual Tracking

Ibrahima J. Ndiour and Patricio A. Vela

Georgia Institute of Technology

Dec 16, 2009

Prior related work

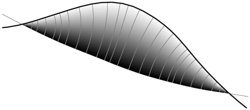
1. M. Niethammer, P.A. Vela, and A. Tannenbaum. *Geometric observers for dynamically evolving curves*. TPAMI 2008.
2. J.D. Jackson, A.J. Yezzi, and S. Soatto. *Tracking deformable moving objects under severe occlusions.* CDC 2004.
3. S. Dambreville, Y. Rathi and A. Tannenbaum. *Tracking deformable objects with unscented Kalman filtering and geometric active contours.* ACC 2006.
4. D. Cremers. *Dynamical statistical shape priors for level set-based tracking*. TPAMI 2006.
5. M. Niethammer, A. Tannenbaum and S. Angenent. *Dynamic active contours for visual tracking.* Transactions on Automatic Control 2006.
6. N. Vaswani, Y. Rathi, A. Yezzi and A. Tannenbaum. *Deform PF-MT: particle filter with mode tracker for tracking non-affine contour deformations.* Transactions on Image Processing 2009.

- Gain selection: currently, most filtering designs proposed for tracking consider a fixed gain, manually specified.

Abstract

- This work examines the task of closed curve filtering for segmentation-based visual tracking.
- We discuss the derivation of a local, linear description for planar curve variation and curve uncertainty. Subsequently, a simple, locally optimal filtering procedure is derived.
- The principal contribution is the derivation of a mechanism for estimating the optimal gain associated to the curve filtering process for planar curves, given quantitative uncertainty levels.
- Experiments were conducted to validate the proposed method and resulting observer design.

- *Correspondence trajectories:*
  Solve the Laplace equation $\Delta u = 0$ (with boundary conditions) to obtain a harmonic field. The corresponding characteristic vector field is given by $\frac{\nabla u}{||\nabla u||}$.
- *Curve coordinate system:*
  Following the distance characteristics starting at a curve point defines the local transverse coordinate system.

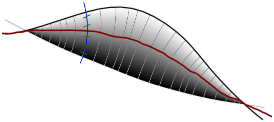


IEEE Celebrating 125 Years *of Engineering the Future*

Georgia Tech

Transverse curve coordinates

- *Correspondence trajectories:*
  Solve the Laplace equation $\Delta u = 0$ (with boundary conditions) to obtain a harmonic field. The corresponding characteristic vector field is given by $\frac{\nabla u}{||\nabla u||}$.
- *Curve coordinate system:*
  Following the distance characteristics starting at a curve point defines the local transverse coordinate system.



Celebrating 125 Years
of Engineering the Future

Curve filtering and the optimal gain

- Consider estimates $\hat{\mathcal{C}}$, $\hat{\mathcal{C}}^-$ of the true curve $\mathcal{C}$ and a measurement $\mathcal{C}^m$.
- Assume the measurement is independent from the prediction, i.e. $\mathrm{Cov}(\mathcal{C}^{\mathrm{m}}, \hat{\mathcal{C}}^-) = 0$.
- In point notation, the curve errors of the estimates are given by:

$$\begin{cases} \hat{e}^-(s) & = \hat{x}^-(s) - x(s) \\ \hat{e}(s) & = \hat{x}(s) - x(s) \end{cases}$$

- The variance associated with the errors is:

$$\begin{cases} P^-(s) & = E\left([\hat{x}^-(s) - x(s)]^2 \right) > 0 \\ P(s) & = E\left([\hat{x}(s) - x(s)]^2 \right) > 0 \end{cases}$$

Curve filtering and the optimal gain

- Consider estimates $\hat{\mathcal{C}}$, $\hat{\mathcal{C}}^-$ of the true curve $\mathcal{C}$ and a measurement $\mathcal{C}^m$.
- Assume the measurement is independent from the prediction, i.e. $\mathrm{Cov}(\mathcal{C}^{\mathrm{m}}, \hat{\mathcal{C}}^-) = 0$.
- In point notation, the curve errors of the estimates are given by:

$$\begin{cases} \hat{e}^-(s) & = \hat{x}^-(s) - x(s) \\ \hat{e}(s) & = \hat{x}(s) - x(s) \end{cases}$$

- The variance associated with the errors is:

$$\begin{cases} P^-(s) & = E\left([\hat{x}^-(s) - x(s)]^2\right) > 0 \\ P(s) & = E\left([\hat{x}(s) - x(s)]^2\right) > 0 \end{cases}$$

IEEE Celebrating 125 Years of Engineering the Future

Georgia Tech

Curve filtering and the optimal gain

- Consider estimates $\hat{\mathcal{C}}$, $\hat{\mathcal{C}}^-$ of the true curve $\mathcal{C}$ and a measurement $\mathcal{C}^m$.
- Assume the measurement is independent from the prediction, i.e. $\mathrm{Cov}(\mathcal{C}^{\mathrm{m}}, \hat{\mathcal{C}}^-) = 0$.
- In point notation, the curve errors of the estimates are given by:

$$\begin{cases} \hat{e}^-(s) & = \hat{x}^-(s) - x(s) \\ \hat{e}(s) & = \hat{x}(s) - x(s) \end{cases}$$

- The variance associated with the errors is:

$$\begin{cases} P^-(s) & = E\left(\left[\hat{x}^-(s) - x(s)\right]^2\right) > 0 \\ P(s) & = E\left(\left[\hat{x}(s) - x(s)\right]^2\right) > 0 \end{cases}$$

IEEE Celebrating 125 Years of Engineering the Future

Georgia Tech

Curve filtering and the optimal gain (2)

- Further assume the measurement error variance ($R(s) > 0$) varies smoothly with s.
- We seek an optimal selection of the gain K so that the error covariance $P(s)$ is minimized under the update model:

$$\hat{x}(s) = \hat{x}^{-}(s) + K(\hat{x}^{m}(s) - \hat{x}^{-}(s))$$

- The setup reduces the problem of finding the optimal selection of K to a one-dimensional problem, for which the optimal choice of K is given by the Kalman gain:

$$K = P^{-}\left(P^{-} + R\right)^{-1}$$

- The associated error variance is:

$$P = P^{-}\left(1 - K\right)$$

IEEE Celebrating 125 Years of Engineering the Future

Georgia Tech

Curve filtering and the optimal gain (2)

- Further assume the measurement error variance ($R(s) > 0$) varies smoothly with s.
- We seek an optimal selection of the gain K so that the error covariance $P(s)$ is minimized under the update model:

$$\boxed{\hat{x}(s) = \hat{x}^{-}(s) + K(\hat{x}^{m}(s) - \hat{x}^{-}(s))}$$

- The setup reduces the problem of finding the optimal selection of K to a one-dimensional problem, for which the optimal choice of K is given by the Kalman gain:

$$K = P^{-}\left(P^{-} + R\right)^{-1}$$

- The associated error variance is:

$$P = P^{-}\left(1 - K\right)$$

IEEE Celebrating 125 Years of Engineering the Future

Georgia Tech

Curve filtering and the optimal gain (2)

- Further assume the measurement error variance ($R(s) > 0$) varies smoothly with s.
- We seek an optimal selection of the gain K so that the error covariance $P(s)$ is minimized under the update model:

$$\hat{x}(s) = \hat{x}^{-}(s) + K(\hat{x}^{m}(s) - \hat{x}^{-}(s))$$

- The setup reduces the problem of finding the optimal selection of K to a one-dimensional problem, for which the optimal choice of K is given by the Kalman gain:

$$\boxed{K = P^{-}\left(P^{-} + R\right)^{-1}}$$

- The associated error variance is:

$$P = P^{-}\left(1 - K\right)$$

IEEE Celebrating 125 Years of Engineering the Future

Georgia Tech

Curve filtering and the optimal gain (2)

- Further assume the measurement error variance ($R(s) > 0$) varies smoothly with s.
- We seek an optimal selection of the gain K so that the error covariance $P(s)$ is minimized under the update model:

$$\hat{x}(s) = \hat{x}^{-}(s) + K(\hat{x}^{m}(s) - \hat{x}^{-}(s))$$

- The setup reduces the problem of finding the optimal selection of K to a one-dimensional problem, for which the optimal choice of K is given by the Kalman gain:

$$K = P^{-}\left(P^{-} + R\right)^{-1}$$

- The associated error variance is:

$$\boxed{P = P^{-}\left(1 - K\right)}$$

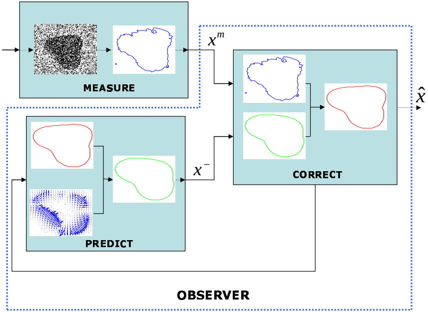
IEEE Celebrating 125 Years of Engineering the Future

Georgia Tech

Extension to visual tracking

The filtering strategy is now applied to visual tracking.

- The target motion is decomposed under a rigid motion component (group) and a non-rigid motion component (shape).
- The combination of the shape filtering strategy with a group filtering strategy and a dynamical model results in an observer for visual tracking.



Dynamical prediction model

The dynamical prediction model describes the state evolution based on prior knowledge of the target movement.

- For the group variable, one can assume a constant velocity model:

$$\dot{g} = \xi, \qquad \dot{\xi} = 0$$

- For the contour, multiple prediction strategies can be considered:
  - *constant curve deformation:*

$$\dot{\Psi} = \nabla \Psi \cdot \Theta \qquad \dot{\Theta} = \nabla \Theta \cdot \Theta$$

  - *dynamic elastic prior:*

$$\mathcal{C}_t = \beta \mathcal{N}, \quad \beta_t = \frac{1}{2}\beta^2 \kappa$$

Dynamical prediction model

The dynamical prediction model describes the state evolution based on prior knowledge of the target movement.

- For the group variable, one can assume a constant velocity model:

$$\boxed{\dot{g} = \xi, \qquad \dot{\xi} = 0}$$

- For the contour, multiple prediction strategies can be considered:
  - *constant curve deformation:*

$$\dot{\Psi} = \nabla\Psi \cdot \Theta \qquad \dot{\Theta} = \nabla\Theta \cdot \Theta$$

  - *dynamic elastic prior:*

$$\mathcal{C}_t = \beta\mathcal{N}, \quad \beta_t = \tfrac{1}{2}\beta^2\kappa$$

Celebrating 125 Years
of Engineering the Future

Dynamical prediction model

The dynamical prediction model describes the state evolution based on prior knowledge of the target movement.

- For the group variable, one can assume a constant velocity model:

$$\dot{g} = \xi, \qquad \dot{\xi} = 0$$

- For the contour, multiple prediction strategies can be considered:
  - *constant curve deformation:*

$$\dot{\Psi} = \nabla\Psi \cdot \Theta \qquad \dot{\Theta} = \nabla\Theta \cdot \Theta$$

  - *dynamic elastic prior:*

$$\mathcal{C}_t = \beta\mathcal{N}, \quad \beta_t = \frac{1}{2}\beta^2\kappa$$

Celebrating 125 Years
of Engineering the Future

Dynamical prediction model

The dynamical prediction model describes the state evolution based on prior knowledge of the target movement.

- For the group variable, one can assume a constant velocity model:

$$\dot{g} = \xi, \qquad \dot{\xi} = 0$$

- For the contour, multiple prediction strategies can be considered:
  - *constant curve deformation:*

$$\dot{\Psi} = \nabla\Psi \cdot \Theta \qquad \dot{\Theta} = \nabla\Theta \cdot \Theta$$

  - *dynamic elastic prior:*

$$\mathcal{C}_t = \beta\mathcal{N}, \quad \beta_t = \frac{1}{2}\beta^2\kappa$$

Measurement model

- Measurement of the target can be achieved through any segmentation algorithm applied to the current image.
- A registration procedure can be applied in order to decompose the resulting segmentation into a measurements for the group and a measurement for the curve.
- The velocity field can be measured by computing the optical flow between two subsequent aligned images and projecting it onto the measured curve normals.

Update model

The update model refines the estimate of the observer internal state given state measurements:

- Correction on the group variable can be carried out using finite dimensional filtering update laws (Kalman, EKF or UKF updates).
- Correction on the shape component requires performing:

$$\hat{x}(s) = \hat{x}^{-}(s) + K(x^{m}(s) - \hat{x}^{-}(s)) = Kx^{m}(s)$$

- Correction on the shape velocities is done through:

$$\hat{\beta}(s) = \hat{\beta}^{-} + K_{vx}(x^{m}(s) - \hat{x}^{-}(s)) + K_{vv}(\beta^{m}(s) - \hat{\beta}^{-}(s))$$

IEEE Celebrating 125 Years of Engineering the Future

Georgia Tech

Update model

The update model refines the estimate of the observer internal state given state measurements:

- Correction on the group variable can be carried out using finite dimensional filtering update laws (Kalman, EKF or UKF updates).
- Correction on the shape component requires performing:

$$\hat{x}(s) = \hat{x}^{-}(s) + K(x^{m}(s) - \hat{x}^{-}(s)) = Kx^{m}(s)$$

- Correction on the shape velocities is done through:

$$\hat{\beta}(s) = \hat{\beta}^{-} + K_{vx}(x^{m}(s) - \hat{x}^{-}(s)) + K_{vv}(\beta^{m}(s) - \hat{\beta}^{-}(s))$$

IEEE Celebrating 125 Years of Engineering the Future

Georgia Tech

Update model

The update model refines the estimate of the observer internal state given state measurements:

- Correction on the group variable can be carried out using finite dimensional filtering update laws (Kalman, EKF or UKF updates).
- Correction on the shape component requires performing:

$$\hat{x}(s) = \hat{x}^{-}(s) + K(x^{m}(s) - \hat{x}^{-}(s)) = Kx^{m}(s)$$

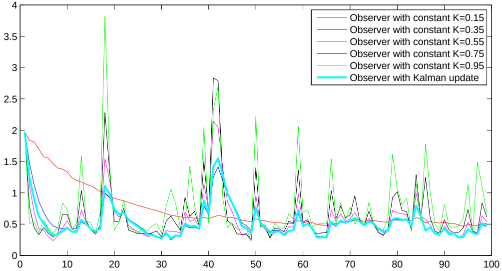
- Correction on the shape velocities is done through:

$$\hat{\beta}(s) = \hat{\beta}^{-} + K_{vx}(x^{m}(s) - \hat{x}^{-}(s)) + K_{vv}(\beta^{m}(s) - \hat{\beta}^{-}(s))$$

Celebrating 125 Years
of Engineering the Future

Experiments: gain optimality

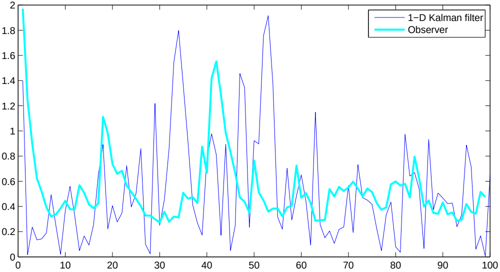
- Segmentations on a static sequence of images corrupted with noise were generated and then filtered using the proposed local filter.
- A modified version of the filter was also used with a fixed gain strategy. A gain sweep from 0.05 to 0.95 in 0.05 gain increments was performed to verify if the Kalman gain converged to its optimal value.



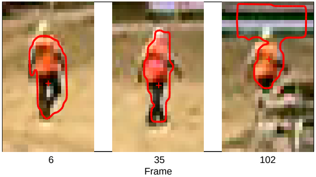
Celebrating 125 Years
of Engineering the Future

Experiments: comparison to an actual 1D system

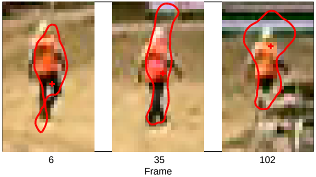
- Several 1D Kalman filter simulations were run to compare the filter simulations against a true 1D system. Here, we depict the evolution of the error for a 1D system and for a simulated static tracking scenario (no group motion, no curve deformation).



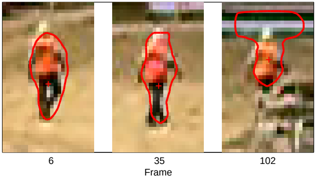
Experiments: Visual tracking



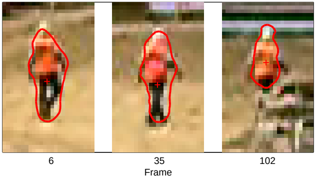
(a) Active Contour



(b) Shape-constrained



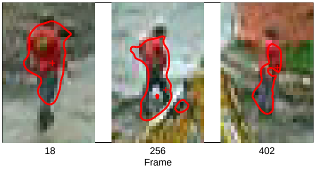
(c) Deformotion Filter



(d) Local Kalman

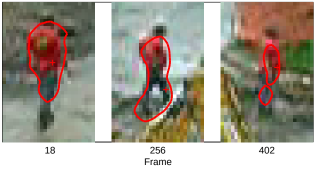
Experiments: Visual tracking

Experiments: Visual tracking (2)

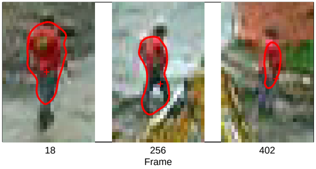


(a) Active Contour

(b) Shape-constrained



(c) Deformotion Filter



(d) Local Kalman

Experiments: Visual tracking (2)

Experiments: quantitative metrics

Metric \ Algorithm	AC	Deformotion	Shape	Observer
Trackpt error (L_2/L_∞)	~~2.2/ 6.6~~	~~2.2 / 9.6~~	~~7.6 / 18.5~~	1.8 / 6.2
NMP (avg/max)	~~78 / 202~~	~~72 / 172~~	~~87 / 160~~	63 / 111
Mean Laplace (avg/max)	~~1.0 / 3.7~~	~~0.9 / 3.1~~	~~1.2 / 2.6~~	0.7 / 1.3
Max Laplace (avg/max)	~~2.9 / 8.9~~	~~2.3 / 7.9~~	~~3.4 / 8.4~~	2.0 / 3.5
# Frames tracked	~~109~~	~~109~~	~~115~~	350

Metric \ Algorithm	AC	Deformotion	Shape	Observer
Trackpt error (L_2/L_∞)	4.4/ 9.4	4.3 / 9.3	~~3.3 / 9.7~~	2.3 / 5.5
NMP (avg/max)	52 / 149	49 / 125	~~98 / 195~~	48 / 95
Mean Laplace (avg/max)	1.2 / 14.9	1.1 / 18.4	~~2.7 / 8.4~~	0.8 / 2.4
Max Laplace (avg/max)	3.0 / 18.3	2.7 / 21.5	~~8.6 / 18.9~~	2.3 / 6.4
# Frames tracked	1014	1014	~~430~~	1014

Celebrating 125 Years
of Engineering the Future

Thank you for your attention.

Questions?

IEEE
Celebrating 125 Years
of Engineering the Future

Georgia Tech